Portfolio Construction and Global Asset Allocation:

A Practitioner Solution to a Black-Litterman Flaw

"Worldly wisdom teaches that it is better to fail conventionally than to succeed unconventionally."

John Maynard Keynes.

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Abstract

This paper presents a variation of the Black-Litterman Model (B&L) for portfolio construction and global asset allocation practices. The methodology proposed retains the Bayesian approach of the original B&L model and, in particular, the derivation of the Posterior Vector of expected market returns incorporating subjective investors' views. The variation from the canonical model we are presenting is in the construction of the Prior Equilibrium Vector of implied market return (B&L Market Equilibrium Starting Point), which we derive without the help of questionable models (e.g. the CAPM), without restrictive and subjective assumptions, without the need to pre-determine an all-encompassing investment universes/global benchmarks, and without forecast. Our version of the vector of market implied expected returns provides an unbiased and more robust equilibrium starting point, thus generating superior portfolios as measured by the Sharpe Ratio when applied to the Black-Litterman construct to determine the Posterior Vector. In addition, our methodology can start from any subjective investment universe, thus completely removing the logistical hurdle of defining a global universe.[2]

Introduction

There are two essential and equally crucial elements of successful portfolio management: idea origination and portfolio construction or asset allocation; everything else is detail. Despite this obvious and universal statement, if one quizzes investors on how they pick and assess their portfolio managers, chances are one will hear a litany of superlatives (or diminutives as the case

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may be) on the first element; but very little on the second one. One will almost never hear "…my manager is excellent at portfolio construction".

The reason boils down to the simple fact that portfolios by construction are risk management tools – otherwise one would invest everything into the higher promising investment. Risk management in turn is the science (some would call it the art) of measuring and forecasting risk; but risk is by its very nature and definition random, i.e. unpredictable! Hence, it follows that the practice or theory of portfolio construction has tended to fall much more under the auspice of the *mad statistician* instead of the ingenious investment manager. To be sure, there is no question that every diligent manager will take risk into consideration when assessing an individual investment; but when it comes to sizing the position in the portfolio and mixing it with other investments, the tendency will be to fall back on statistical practice and theories, e.g. Beta, Risk Budgeting, etc. That is to say, when it comes to portfolio construction we essentially rely on past history[3], i.e. we are *still driving by looking in the rear view mirror*. By attempting to forecast risk rather than expected return, it would seem that portfolio construction is mostly concerned with predicting the unpredictable, that is: volatility…a very poor proxy for true risk![4]

1. Modern Portfolio Theory

The seed of this state of affairs was sown in 1952 by Nobel Laureate Harry Markowitz, after he realized that stock market theory lacked an analysis of the role of risk. While searching for a solution, Markowitz reportedly "stumbled" onto a statistical book opened to the page on correlation. What he read was simple and revealing: *the combined variance of two or more variables will be lower than the sum of the variance of each variable, as long as the correlation amongst the variables is less than 1*. That is, by choosing securities that do not 'move' exactly together, Markowitz showed that risk can be reduced for the same return. This ingenious insight led to the development of his seminal Theory of Portfolio Allocation under Uncertainty, published in 1952 in *the Journal of Finance*. Hence, *Modern Portfolio Theory (MPT)* as it is now known was born out of a well-tested statistical axiom. Indeed, this concept was at the base of further research by W. Sharpe, Trevor, Fama & French, and many others, leading ten years later to the introduction of the equally seminal *Capital Asset Pricing Model* (CAPM) and factor models.

MPT received a further boost in the late 80s, when Gary P. Brinson, Randolph Hood, and Gilbert L. Beebower (known collectively as BHB) set out to analyze the impact of asset allocation policy on pension plan returns, a topic that had been largely ignored until then. In their seminal paper, "Determinants of Portfolio Performance" published in 1986 in the *Financial Analysts Journal*, BHB asserted that asset allocation is the primary determinant of a portfolio's return variability. Their study compared the returns of 91 large U.S. pension fund allocation to various asset classes with a hypothetical fund holding the same allocation across assets but invested in passive indexes.

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[3] As Warren Buffet has most famously said" *…if history is all there was to the game, librarians would be the richest people in the world".*

[4] Volatility is not risk; permanent loss of capital is!!!

Electronic copy available at: https://ssrn.com/abstract=2864954

Using simple regression analysis, BHB concluded that over the long term (they used 1974 to 1983, so approximately 10 years), asset allocation explained 93.6% of the *variation* in a portfolio's quarterly returns. The controversial implication was that security selection and market timing (i.e. active management and bottom up security "picking") played a minor if not useless roles in generating long term performance!

Given the conclusion, the paper was obviously provocative and was the subject of a heated debate in the industry. In 1997 William Jahnke published a critique of the BHB study, in which he argued: "*The fundamental problem with BHB's analysis is its focus on explaining return volatility (sigma) rather than portfolio returns (mu). In fact, investors should be more concerned with the range of likely outcomes [*event probability, ndr*] over their investment planning horizon than the volatility of returns"; i.e.* we should go back to driving looking straight ahead instead of the rearview mirror! Jahnke went on to warn: "*Fixed asset allocation solutions are inferior to analytically linking forward-looking strategic asset allocation solutions. As the investor's circumstances or market opportunities change (markets are dynamic, irrational at times and certainly not efficient as in Fama's definition), so also should the investor's asset allocation."* In simple words, Jahnke argued to care more about value and/or potential returns, instead of the volatility experienced while the portfolio is "at work".

Regardless of the merit of each side of the debate, no one argued on the actual mathematical techniques used to construct the portfolio. Indeed, asset allocation – dynamic, strategic, tactical or not - relies on the same precept put forward by Harry Markowitz in that a mix of asset classes that are not perfectly correlated will diversify risk and generate optimal, risk-adjusted returns. The question then becomes: can one count on stationary correlation over an investment cycle?

2. Mean Variance Optimizer and Efficiency Frontier

The practical starting point of MPT is that, in a world where returns are normally distributed (bell-shaped), the attributes of each asset are described by the first two moments of the statistical distribution, *mean* and *variance*. A portfolio is then considered **efficient** if for any given level of risk, return is maximized, or if for any given level of return risk is minimized. All efficient portfolios then will sit on a line called the ***Efficient Frontier***; it starts at the minimum variance portfolio and ends at the maximum return portfolio, or at the asset of maximum return. Risk is minimized (diversified) and portfolios are efficient.

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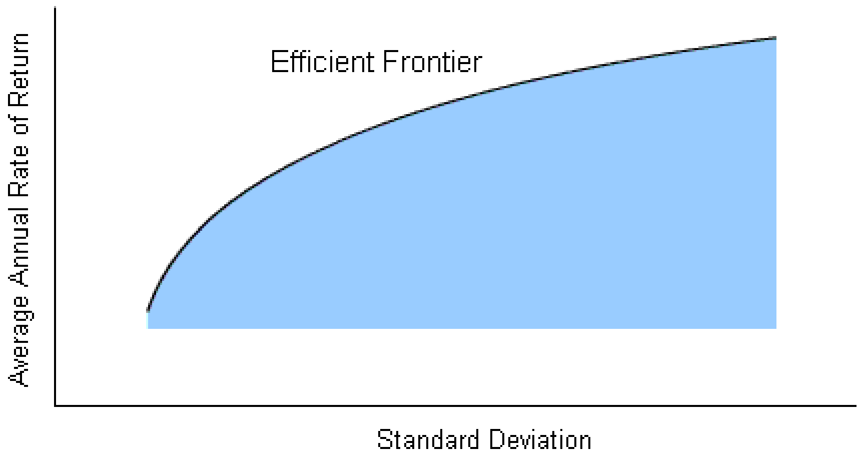


While there is no doubt about the theoretical strength and robust foundation of the Markowitz principle, there are several complications when putting his MPT methodology in practice. It is a perfect example of *normative finance,* or the way finance should be, *positive finance*, the way finance is…and the art of managing money, or that *unquantifiable dose of subjective genius which has troubled and continues to agitate many quant managers.*

The quantitative procedure at the base of Markowitz's model is known as the Mean Variance Optimization (**MVO**) in that portfolios are optimized as a tradeoff of return relative to risk and vice versa. The input for the MVO exercise is (a) the vector of expected returns for each asset (or security), and (b) the Variance Covariance matrix for the entire set of assets. The main output of this tool is what is known as the *Efficiency Frontier*, or the set of portfolios with expected return greater than any other with the same or lesser risk, and lesser risk than any other with the same or greater return. A useful feature of the single period MVO problem is that it is solvable by the quadratic programming algorithm, thus providing a set of weights defining a mean variance efficient portfolio via a simple closed-form solution. The trouble is that despite the theoretical rigor, the results of the MVO can be very deceiving and outright impractical to implement:

1. To start with, although the covariance (input #2 in the MVO) of several assets can be adequately estimated, it is challenging to come up with reasonable estimates of expected returns…other than totally subjective ones of course. The dilemma is that the optimal portfolio calculated via an MVO is extremely sensitive to the input and in particular to the vector of estimates of expected returns. Thus the resulting portfolio, while optimal in the mean-variance space, is highly unstable in that a small change in the input may trigger a disproportionate change in the weights. Clearly, and in addition, the higher the frequency of rebalancing, the larger the reallocation costs, and the lower the benefit of the entire exercise.
2. Second, because of the mechanical work of the Optimizer, the resulting optimal portfolio may not be intuitive and, in some cases, may result in corner-type solutions. For example, when unconstrained, MVO may generate portfolios with large long and short positions;

and, when subject to a long only constraint, portfolios that are concentrated in a small number of assets (one?!). Normally, the typical solution in this case is to impose constraints on the size of each position (min, max); but these constraints are obviously arbitrary and subjective in nature, thus totally rejecting the rigor of the exercise.

3. Lastly, estimate error maximization. Because the inputs are statistical estimates from an unknown population (typically created by analyzing historical data), they cannot be devoid of error. This estimate error typically leads to overinvestment in some asset classes and underinvestment in others.

These three related and well-known practical problems are the reason why mean variance optimization has had limited large scale adoption, despite continuing to provide the theoretical framework for optimal portfolio construction. Prof. Markowitz has often addressed most of these criticisms[5]; and he often concludes that his point remains that, rationally, one *does not put all of one's eggs into one basket.* However, the predicament remains that the MVO at times will tell the investors exactly the contrary; i.e. put all the eggs into one or two baskets (also called corner solutions)!

3. The Black- Litterman Approach

In the early 90s, Fisher Black and Robert Litterman (B&L) of Goldman Sachs, developed a model (known by their names) which attempted to overcome the MVO shortcomings. While their model was introduced as an Asset Allocation method, it is in fact a technique to estimate a vector of expected returns as implied by the market, which is then *tilted* by investor subjective views. This "mixed" vector (market implied plus subjective views) is then used as the input and driver of an MVO exercise[6]. The innovative approach was to improve the input of the MVO, instead of improving the output. Hence, instead of trying to "sanitize" the MVO output by using palliatives such as subjective constraints or worse yet elaborate but equally ineffective mathematical techniques, B&L focused on developing a rigorous and methodological system to define the vector of expected return, or the driver of the MVO. The implication was that a neutral "*equilibrium*" estimate would generate more stable and intuitive results, without having to rely on gimmicks or band aids to subjectively adjust the final portfolio. As B&L put it "… [Global CAPM] *equilibrium returns for equities, bonds and currencies provide neutral starting points for estimating the set of expected excess returns needed to drive the portfolio optimization process. This set of neutral weight can then be tilted in accordance with the investor's view"*[7]*.* Note that they explicitly mention equilibrium "*returns*" which they must determine.

The innovation of the B&L model was twofold:

[5] See "Crisis", Markowitz on the 2008 Global Financial Crisis

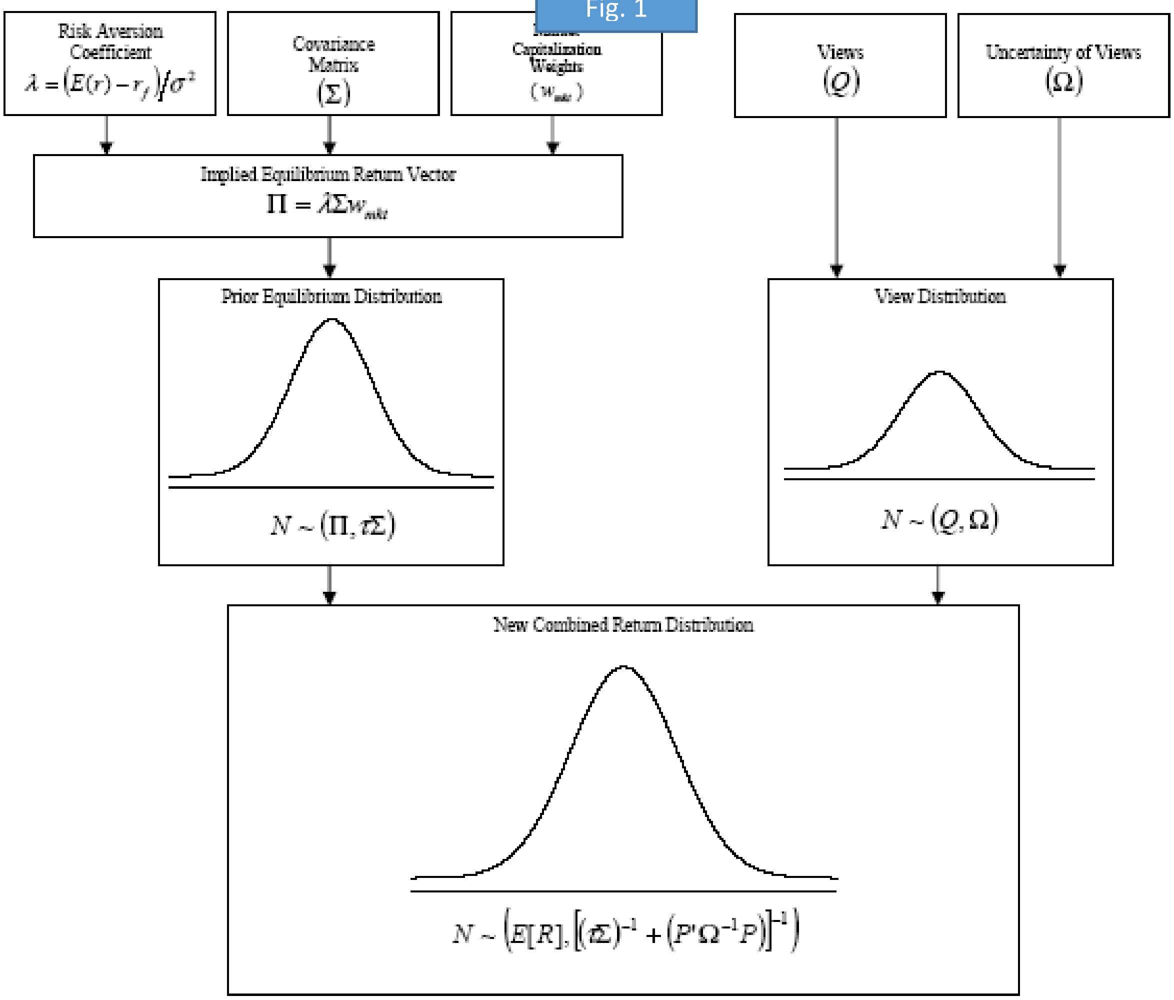
[6] As Fisher Black himself has later empirically proved in a different paper, MVO is typically not that sensitive to small variation of the Variance-Covariance matrix.

[7] Black-Litterman – 1992.

1. First, use the CAPM paradigm and reverse optimization techniques to define a method to extract from the market the Implied Equilibrium Vector of Excess Returns based on market capitalization weights.
2. Then, apply an econometric technique called *Theil Mixed Estimation*[8] that allowed the authors to rigorously combine the market Implied Equilibrium vector previously determined with investors' subjective views.

The following flow chart is a popular graphical representation of the key steps of the B&L Model:

Fig. 1



The methodology (in particular Theil's techniques) is a typical Bayesian approach in that the Implied Equilibrium vector is the *Prior*, while the final vector incorporating (mixing) the subjective views with the *Prior* becomes the *Posterior* vector. Thus, in the B&L construct the investors' views determine the conditional distribution of the vector of input (***E[R]***) in to the MVO (See Fig.1).

[8] See *Principles of Econometrics* – Henry Theil – J. Wiley & Sons. 1971 – pgs. 347-352.

Because of its intuitive approach and the more stable results compared to the MVO, the B&L Model has gained wide application and adoption across the broad spectrum of the industry; this trend has been reinforced recently in light of the *Fintech* boom and the increasing popularity of *Robo Investment*[9]. Indeed, the most popular aspect of the model is the flexibility to combine a market equilibrium starting point with the investor's subjective views on all or a subset – however small - of the investment universe; the investors' views are allowed to be partial (on just a few assets) or complete (on all assets). *This is crucial as portfolio can be constructed by incorporating forward-looking "views" as opposed backward-looking "statistics"*. As Figure 1 shows, however, the model can be theoretically and practically demanding as it is a collection of several diverse tools and theories: CAPM, Reverse Optimization, Theil's Mixed Estimates, Bayes Rules, and MVO. Thus one needs the manager to work closely with the quant, or the manager to be an "*active quant*", almost an oxymoron in today's world. Before moving on, we must describe the B&L model. *Readers may want to skip to section 6 to by-pass the mathematical derivation of the model.*

4. The B&L Formula and its Derivation

The starting point of the B&L model is the usual Quadratic Utility function which is at the base of Markowitz MPT and the CAPM. In condition of general equilibrium, every rational investor[i] will want to maximize the following utility:

(1) $\boldsymbol{MAX_w}$ **: E (** $\boldsymbol{r_p - r_f}$ **) - Λ** $\boldsymbol{\sigma^2_{rp}}$ *where* $\boldsymbol{r_p}$ *and* $\boldsymbol{\sigma^2_{rp}}$ *are return and variance of the portfolio*

(1) in matrix format is equal to*:*

(2) $\boldsymbol{w^T (\mu - r_f) - \Lambda\, w^T\, \Sigma\, w}$ *where **Λ=**risk aversion; **Σ=** VarCoVar matric; **W=**weights*

That is, every rational investor will want more return and less risk, the later proxy by the variance of returns. As long as **Λ ≥ 0**, utility goes up with excess return but will go down with risk.

In order to find the optimal set of weights that maximize the utility, we must differentiate equation (2) with respect to ***w*** and set it equal to zero:

(3) $$\frac{dU}{dw} = (\mu - r_f) - \Sigma z = 0 \qquad \text{where } Z = 2\Lambda w$$

Re-arranging and multiplying each side by S^{-1}, then

(4) $$Z = \Sigma^{-1} (\mu - r_f) \qquad \text{and} \qquad w = \frac{z}{1^T z}$$

where ***W*** is the vector of weights maximizing the Utility function.

[9] Indeed, there has been an abuse of the model in that several Robo "shops" claim to be basing their methodology on the B&L model…a very questionable assertion for the expert eye.

The derivation above is theoretically precise. The problem however is that in practice the inputs for the formula are not observable. Typically, this obstacle is overcome using two main alternatives/options to derive $\boldsymbol{\mu}$**,** or the vector of returns:

- Use historical data. The argument is that the average estimate error incurred by using an historical sample will over time be equal to zero; that is:

$$\boldsymbol{E(r) = \mu + \varepsilon} \qquad \text{where} \quad \boldsymbol{\varepsilon \sim N(0,\, \sigma_\varepsilon^2)} \quad \text{and finally} \quad \mathbf{E(r)} = \boldsymbol{\mu}$$

- Use models to estimate returns, the main one being the CAPM:

$$\boldsymbol{\mu = R_f + \beta * RP}$$

Where:

R_f *is the risk free rate*; β *is estimated via a regression and RP is the Risk Premium*

And here is where B&L departs radically from the traditional MPT approach. Instead of using one of the two alternatives listed above, the authors apply a Bayesian approach which will allow the application of “degrees of belief” or “confidence” on the expected returns, thus providing a marked improvement over the MVO traditional input. Bayesian statistics requires the specifications of the *Prior Distribution* for the unknown variables. In order to determine the Prior B&L utilize the CAPM and its well-known set of underlying assumptions.[10]

According to the CAPM each investor, depending on her/his individual risk tolerance, will allocate a portion of wealth to an optimal portfolio (mean-variance efficient) and the remainder to risk-free lending or borrowing. All investors will hold risky assets in the same relative proportions, given that every investor expects the same return. For the market to be in (general[11]) equilibrium, the expected return of each asset must be such that investors collectively decide to hold exactly the available supply. If investors all hold risky assets in the same proportions, those proportions must be the proportions in which risky assets are held in the market portfolio—i.e. *the portfolio comprised of all available shares of each risky asset.* In equilibrium, therefore, the optimal portfolio of risky assets must be the market portfolio. [Notice the somewhat circular argument which has nevertheless been accepted for years.]

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[10] First, investors are risk averse. Second, capital markets are perfect in several senses: all assets are infinitely divisible; there are no transactions costs, short selling restrictions or taxes; information is costless and available to everyone; and all investors can borrow and lend at the risk-free rate. Third, investors all have access to the same investment opportunities. Fourth, investors all make the same estimates of individual asset expected returns, standard deviations of return and the correlations among asset returns.

[11] Meaning that every other sub portfolio will also be in equilibrium.

Consequently, B&L postulate that if the optimal portfolio is the market portfolio, then the vector ***w*** of equation (2) no longer needs to be calculated in equation (4) above as the *weights of the optimal portfolio are observable and are equal to the market cap weights*. Using reverse optimization, B&L then turn the problem around and solve equation (2) for the vector of excess return[12]:

(5) $$(\mu - r_f) = 2\Lambda\Sigma w$$

Σ and ***w*** are known; **Λ** needs to be determined. If the market is Mean Variance efficient, then B&L derive the following formula for Lambda, or the price of risk implied in the market:

(6) $$\Lambda = \frac{E(r_m - r_f)}{2\sigma_m^2}$$

Substituting it in **(5):**

(7) $$\Pi = \frac{E(r_m - r_f)}{\sigma_m^2}\Sigma w \quad \text{or} \quad \Pi = \Lambda\Sigma w$$

where **∏** is the Implied Equilibrium Vector of Excess Return, or the ***Prior*** in the Bayesian terminology. Thus, the "starting point" has been defined.

5. Formulating Subjective Views

Having defined the "natural starting point" **∏**, B&L at this point set up the framework to capture the subjective views. To be sure this is the most complex and abstract part of the model, and the part that has spawned several variations of the original model[13]. Surprisingly however, very little has been written on the validity of the determination of the Implied Equilibrium Vector **∏,** or the crucial starting point! The suitability of this vector as determined by B&L is precisely the objective of this paper. But first we must outline the rest of the B&L model.

[12] NB: The vector of CAPM returns is the same as the vector of reverse optimized returns when the CAPM returns are based on implied betas relative to the market capitalization-weighted portfolio.

[13] See Idzorek. Op. Cit. and J. Walters, Op. Cit.

The subjective views are integrated in the model using three elements: a vector of views ***Q***, a pick (or link) matrix ***P***, and a variance covariance matrix of the error term of the views ***Ω***.

The vector of views ***Q*** is a *K x 1* column vector where *K* is the number of subjective views:

$$\boldsymbol{Q}_{(k\,x\,1)} = \begin{bmatrix} q_1 \\ \cdots \\ q_k \end{bmatrix} \quad \text{with } \mathbf{0} \geq \boldsymbol{K} \leq \boldsymbol{n}$$; and where ***n*** is the number of assets.

Note that the investors do not need to have a view on each and every asset entering the allocation exercise; indeed, this is a major attractive aspect of the B&L model relative to traditional MVO (in MVO investors need to have a view on every single asset). A special case of this feature is when the investor has no view at all, i.e. ***K*** = 0; in this case B&L recommend that the investor buys the market portfolio.

Normally, each view in the vector ***Q*** is expressed in percentage term and may represent a relative or absolute view. For example, if the investor has three views, two relative and one absolute:

- View 1 (relative): Emerging Equity will outperform European Equities by 10%.
- View 2 (absolute): Bunds will return 4%.
- View 3 (relative): US Small Cap will outperform both US Large Cap and Global Equities by 5%.

The vector of views will then be a (3x1) vector and look as follows:

$$\boldsymbol{Q} = \begin{bmatrix} \mathbf{0.10} \\ \mathbf{0.04} \\ \mathbf{0.05} \end{bmatrix}$$

Each view expressed in vector ***Q*** must then be matched (*linked)* to specific assets (or the assets affected by the view); this is done via a (link) matrix ***P***. Matrix ***P*** will have a *k x n* dimension:

$$\boldsymbol{P}_{(k\,x\,n)} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{k,1} & \cdots & p_{k,n} \end{bmatrix}$$ where ***K*** is the number of views, and ***n*** is the number of assets

There are different versions in the literature to populate the matrix ***P.*** Litterman[14] uses percentage value for the assets affected by the views. Idzorek[15] uses a market capitalization weighting scheme whereby "…*the relative weighting of each asset is proportional to the asset's market cap divided by the total market capitalization of either the outperforming or underperforming assets of that particular view*".

[14] Litterman - 2003, pag.82.
[15] Idzorek - 2002, pag.12.

Satchell and Scowcroft[16] suggest instead an equal weighting scheme which is actually easy to implement and is reportedly the most utilized in practice. According to this method, each row of Matrix ***P*** representing a relative view must be equal to zero (the sum of all element in that row that is must be = 0), while each row representing an absolute view must be equal to one. For example, if there are 8 assets (***n*** = 8), and three views, as per the example for ***Q*** above (two relative and one absolute), then the matrix P will have the following format:

$$\boldsymbol{P}_{(3\ x\ 8)} = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -0.5 & 0 & 1 & 0 & -0.5 & 0 & 0 & 0 \end{bmatrix}$$

(3 views and 8 assets).

The first row should be read as follows: the investor believes that the asset in the fourth column (where the element of the matrix is +1) will outperform the asset in the second column (where the element is -1). The amount of outperformance is provided by the corresponding first row in the vector of views ***Q;*** in the example of ***Q*** above, 10% (or 0.10). The second row says that the asset in the seventh column (+1) will return an absolute amount of 4% (0.04), or the amount equal to the second row in the vector of views ***Q***. The third row states that the asset in the third column (+1) will outperform the assets in the first (-0.5) and fifth (-0.5) column by 5% (0.05), or the amount equal to the third row of the vector of views ***Q***. The zeros everywhere else mean that there are no views, relative or absolute, on those assets.

Satchell & Scowcroft's method is criticized in that it ignores the market capitalization (size) of the asset(s) affected by the view(s). If two assets listed in Matrix ***P*** are widely different in size – e.g. one is a large cap, and the other is a small cap – and given that both assets are subject to the same weighting method in the matrix ***P***, then the end result of the B&L could potentially generate large tracking errors relative to the benchmarks used to define the market portfolio. While this objection may be reasonable, it assumes inaccurately that there is indeed a well-defined, observable benchmark universally representative for all investors (see below for a more detailed discussion on this point). It also assumes that risk is equal to the deviation from "the" benchmark, altogether an anachronistic view of the investment profession. The fact remains that this equal weight method is intuitive and easy to implement, thus reducing the practical burden of an already complex model.

Having specified the vector of views ***Q*** and the link matrix ***P***, the next element needed to incorporate subjective views in the model is a matrix that quantifies the uncertainties of the views, or a variance of variance-covariance of the views, or the matrix called ***Ω*** (Omega).

To be sure, and as an aside comment, the matrix ***Ω*** together with the scalar ***τ*** (tau, the last remaining variable to define and discussed below) are probably the most controversial, confusing and difficult part of the B&L model. They have directly or indirectly stimulated a prolific set of

[16] Idzorek – Op. Cit.

research and different versions of the B&L Model. Jay Walters of Blacklitterman.Org[17] has actually compiled a thorough review of the several versions of B&L models existing out there, classifying them according to two requirements: points estimate versus distribution estimates, and the inclusion or not of the parameter $\boldsymbol{\tau}$. In order for a model to be "canonical Black-Litterman" it would need to match both requirements. Walters appears to be discarding the various models unless they are pure to the Bayes Rules and the Mixed Estimates Techniques. The dilemma is that most of these models, while intellectually impressive, seem to be bumping each other exclusively on academic ground, forgetting that the B&L Model was not conceived to be a statistical treatise, but instead was developed as a practical methodology to address the fallacies of MPT and MVO, i.e. it was supposed to help the practitioner.

Getting back to the derivation of **Ω**, the basic Black-Litterman model does not provide an intuitive way to quantify this element of the final formula; the authors actually leave it up to the investor to compute the variance of the views **Ω**[18]**.** As a consequence, the derivation of **Ω**, albeit less controversial than **τ**, has generated an equally complex amount of variations from the original model. The starting point is that a view has the form **Q + ε**, or the view plus an error term ***epsilon***. The uncertainty of the views results in a random, unknown, independent, normally-distributed Error Term Vector (**ε**) with a mean of ***0***, a variance(**ω**), and a covariance matrix **Ω**. The variances of the error terms (**ω,** i.e. the diagonal element of **Ω**) represent the uncertainty of the views, thus the larger the variance of the error term (**ω**), the greater the uncertainty of the view.

The authors' definition of the variance covariance matrix **Ω** is that the off diagonal elements should be equal to zero, meaning that each subjective view should be (statistically) independent; a necessary theoretical prerequisite, but unfortunately difficult to realize in practice. Indeed, and in order to be in line with Theil's Mixed Estimation[19] - or the technical base of the B&L model - the combination of the investor's view actually defines the conditional distribution. This requires that each view must be independent, which means in turn that the Variance Covariance matrix of the view (**Ω**) must be diagonal, i.e. the covariance between views is zero. Note that defined as such, the inverse of Omega, $\boldsymbol{\Omega^{-1}}$ is also known as the confidence level of the investor's views. Omega will have the following format:

$$\boldsymbol{\Omega} \text{ a k} \times \text{k matrix } = \begin{bmatrix} \boldsymbol{\omega_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\omega_k} \end{bmatrix}$$

[17] The Black-Litterman Model In Detail Initial: February 2007 Revised: June 20, 2014 © Copyright 2007-14: Jay Walters, CFA jwalters@blacklitterman.org. J. Walters even came up with a labeling system: Canonical, Alternate, and Beyond Black-Litterman for the B&L.

[18] An intuitive albeit more laborious method, is estimating a confidence interval. The investor can specify the variance using a confidence interval around the estimated mean return, e.g. Asset has an estimated 3% mean return with the expectation it is 68% likely to be within the interval (2.0%, 4.0%). Knowing that 68% of the normal distribution falls within 1 standard deviation of the mean allows us to translate this into a variance for the view of (1%).

[19] E. Theil (1970), Op. Cit.

In practice, there are a couple of alternatives to determine ***Ω*****:**

1. A simpler one which disregards the requirement of having zeros on the off diagonal element of the matrix. Then **Ω** can be determined as follows:

   (8) $$\Omega = \tau P \Sigma P^T$$

2. A more laborious but correct one, where each element on the diagonal of **Ω** (the variances), is calculated separately and the off diagonal are set equal to zero. Thus, the variance of an individual view is

   (9) $$P_k^T \, \Sigma \, P_k$$

   Where P_k is the 1 x N row vector from Matrix ***P*** that corresponds to the kth view and ***Σ*** is the covariance matrix of excess returns.

At this point, the last variable left to define is ***Tau***, in our view the most difficult variable to grasp. Notwithstanding the academic diatribe on this specific element of the model, a simplistic albeit seemingly valid explanation is that the scalar **τ** is there in the B&L formula in order to address a reasonably controversial issue, which is that, since the equilibrium returns are not actually estimated, the estimation error cannot be directly derived. To overcome this hurdle, Black and Litterman made the simplifying assumption that the structure of the covariance matrix of the estimate is proportional to the covariance of the actual equilibrium returns **Σ**. They then created a parameter, **τ**, as the constant of proportionality. And since – the authors posit - the uncertainty in the mean is less than the uncertainty in the return, they use a value close to zero for **τ**, typically 0.025. Satchell and Scowcroft instead set the value at 1[20]. The practical point is that it makes little empirical difference and one can pick and choose the value of **τ** depending on the level of conviction on the methodology used to estimate **∏,** e.g. what benchmark? What market portfolio? Idzorek adds that the scalar **τ** should be inversely proportional to the relative weights given to the Implied Return Vector[21]. Note that it might appear that defining ***Tau*** maybe a case

[20] Because they use point estimates instead of distributions, their model does not include any information on the precision of the estimate. This allows them to recommend setting τ = 1. They also introduce a stochastic τ, but because they use point estimates this really becomes a model with a stochastic covariance of returns.
[21] Idzorek, Op. Cit. Page. 14.

of academic rigor versus practical necessity. Not so; on the contrary it addresses a much bigger problem of the model, or the suitability of the method to derive **∏.**

Having defined the investor's views and stated them as the conditional distribution, B&L then finalize their methodology by applying Theil's Mixed Estimation technique, arriving at the final formula for the combined ***Posterior Vector of Excess Returns E(R):***

$$\textbf{(10)} \qquad E(R) = \left[(\boldsymbol{\tau} \boldsymbol{S})^{-\mathbf{1}} + \boldsymbol{P}^{\boldsymbol{T}} \Omega^{-\mathbf{1}} \boldsymbol{P} \right]^{-\mathbf{1}} \times \left[(\boldsymbol{\tau} \boldsymbol{S})^{-\mathbf{1}} \Pi + \boldsymbol{P}^{\boldsymbol{T}} \Omega^{-\mathbf{1}} \mathbf{Q} \right]$$

The first element of the formula satisfies general conditions required by the statistical technique. The second part is the most intuitive one. It should be seen as the sum of two weighted vectors - **∏** or the equilibrium starting point, and **Q** the vector of subjective views - where the weights are the level of confidence (note: the inverse of the variance matrix) for each vector, or $(\boldsymbol{\tau} \boldsymbol{S})^{-\mathbf{1}}$, and $\boldsymbol{P}^{\boldsymbol{T}} \Omega^{-\mathbf{1}}$. The new *Posterior* vector is then used as the input in a traditional optimizer to calculate an efficient portfolio (Mean-Variance Efficient).

Despite its somewhat inhibiting complexity, the B&L model marks a radical change and a definite improvement over traditional portfolio construction and asset allocation. Indeed, the authors' application of Theil's Mixed Estimation is equally if not more ingenious than Markowitz's application of the academically more modest correlation axiom.

To start with, given the statistical construct and the resulting conditional distribution of the *Posterior* vector, the model will reduce, in some cases markedly the estimation error, thus lowering the probably of corner solutions or highly concentrated portfolios – i.e. one of the three main shortcomings of the MVO mentioned above. Any expert econometrician staring at Fig.1 above will agree with this statement. This alone will trump the constant necessity of the traditional MVO to impose subjective and nonsensical constraints. In addition, while the model remains susceptible to the mechanics of the optimizer (once that the Posterior Vector is defined, it must still be run through a traditional optimizer), the resulting portfolio are much more intuitive as they incorporate and combine subjective views with a market implied natural starting point. Indeed, given the statistical construct, the end portfolios will reflect investors' subjective views and not just mathematical conditions and arbitrary constraints. This "improvement" will address at least in part the second of the three main shortcomings of the MVO listed above.

6. The Fallacy of the CAPM…and its effect on the B&L Model

Despite its success, the B&L model is not infallible and does have some flaws on both academic and practical grounds. As mentioned above, most of the work subsequent to the publication of the original model has focused primarily on enhancing the definitions of views and/or the statistical attributes of the *Posterior Vector*. While there is no doubt that all this body of academic work is indeed relevant and may provide an improvement over the original model, there has been unfortunately little attention paid to the cogency of the derivation of the *Prior Equilibrium Vector* **∏**, or the starting point and therefore the crucial "socket" of the model.

As it were, the entire concept of the *Implied Equilibrium Vector of Excess Returns* at the base of the B&L rests on the CAPM and the crucial assumptions sustaining that theory. Indeed, in order for B&L to extract from the market the implied excess returns (∏) using the CAPM, the authors must assume that the market portfolio (and therefore its weights/market cap) is the optimal portfolio[22]. And in order to do that, the authors must explicitly accept the assumptions at the base of the CAPM, e.g.:

- Investors are risk averse.
- Capital markets are perfect in that:
    - all assets are infinitely divisible;
    - there are no transactions costs;
    - there are no short selling restrictions or taxes;
    - information is costless and available to everyone;
    - all investors can borrow and lend at the risk-free rate as much as they desire.
- Investors all have access to the same investment opportunities.
- Investors all make the same estimates of individual asset expected returns, standard deviations of return and the correlations among asset returns; i.e. all relevant information is public, thus market are semi-strong efficient.

The trouble is that these assumptions are so restrictive that, in the end, they invalidate the final conclusions and results of the model, as the huge body of criticism to the CAPM has attested. In addition, most if not all practitioners will agree that the assumptions above are an oversimplified view of the market, one that may exist only in a theoretical world. In short and as put by Arun Muralidhar of George Washington University the CAPM may be "…*a very special case of a more general Relative Asset Pricing Model*".

Irrespective of the many academic criticisms and refutations of the CAPM, there is plenty of evidence undermining the Model at the empirical and practical level. And here is where the limitation of the CAPM are most relevant (and damaging) for the B&L model in our view.

In particular, note that the CAPM "requires" that all and every one of the risky assets out there must be included in the market portfolio for the market to be in equilibrium – and implied returns being credible. In the 60s when Sharpe et al. were presenting the CAPM, it might have been plausible to conceive that there was indeed such a broadly *identifiable creature* as a market

[22] The CAPM implies that the optimum portfolio is the market portfolio, which lies on the Security Market Line (SML) with a beta factor of one. Individual securities and portfolios with different levels of risk (betas) can be priced because their expected rate of return and beta can be compared with the SML. In equilibrium, all securities will lie on the line, because those above or below are either under or over priced in relation to their expected return. Thus, market demand, or the lack of it, will elicit either a rise or fall on price, until the return matches that of the market.

portfolio consisting of all existing risky securities out there. In the 90s when Black and Litterman were building their model, it had become a huge stretch to accept this same belief, especially since markets were beginning to globalize (that is they were no longer U.S.-centric!). Today, it is downright unreasonable to think that it is not only possible to define or even approximate a [global] portfolio comprising all risky assets, but also to assemble and collect data and attributes on this all-encompassing portfolio. Indeed, the current debate goes even further, arguing about what should be included in the portfolio, e.g. only financial assets? Liabilities? What constitutes an asset? Etc.

The point here is that the theoretical assumption and practical identification of an all-encompassing market portfolio is a major if not crucial assumption of the CAPM, and by reflection the B&L model – at least as far as deriving the Equilibrium Implied Vector of Excess Return (the starting point) is concerned. Essentially, defining the right universe in order to determine the market cap weights and, in turn extract the Implied Returns is a major practical challenge when considering running the B&L model. The question(s) becomes: what should be the right portfolio/benchmark? Is there a (global) benchmark (and market cap weights) representing the entire universe of risky assets out of which one can extract implied market returns reflecting a general equilibrium? Has this benchmark ever been defined to the extent that it satisfies most if not all of the CAPM assumptions and therefore the B&L construct, particularly at the global level which is indeed most relevant nowadays?

B&L attempt to address this major deficiency by claiming the "General Equilibrium" condition, meaning that every sub-portfolio used as the input in their model is also "in equilibrium", again a far stretch from both theory (CAPM) and reality[23]. Litterman, possibly receptive to this major limitation, "confesses" that indeed their model works better when there is a "well-defined" benchmark[24]. J. Walters (staunch supporter of the original model) acknowledges the problem but discards it out of hand stating that typically, portfolio managers are interested in a very specific investment universe[25] (e.g. S&P500, EFEA, MSCI World) and not the entire universe of risky assets…this posturing explanation however does not validate the model. Remember the CAPM requires that" … *investors collectively decide to hold exactly the [entire] available supply…"* of ALL assets out there.

Regardless of these elegant albeit inadequate solutions, the problem is that there hasn't been a proper answer to this theoretical flaw of the CAPM; ditto for its practical and empirical application and extension to other models such as the B&L. Hence, absent both a theoretical and practical satisfactory solution, the derivation of the *Prior Vector* in the B&L model which rests entirely on the CAPM and its assumptions must in our view be questioned or re-addressed.

[23] General Equilibrium would indeed assume that the S&P500 is just as good a representation of the market, portfolio as the MSCI World, or the Barclays' Bond Aggregates.
[24] The Intuition behind the Black-Litterman - 1999
[25] J. Walters. Op. Cit.

7. **Cost of Capital and the Internal Rate of Return…the *True Expected Returns Vector Π***

About twenty years ago, almost at the same time as B&L exposed their model, albeit totally unrelated to their objectives and research, we (the core of Lumen today) were looking for an accurate method to extract from the market an unbiased, comparable across assets, common metric to assess value in order to rank investments across countries, sectors and asset classes. I.e. we were looking for an unbiased market implied expected return. We started the search from the equities, or the base of the capital structure – the idea being that in an efficient world, everything else senior to equity should have a lower implied expected return (i.e. less risk, less return). The obvious (at least to us) theory to fall back on is the most basic law of finance, that is: ***the monetary value of ANY and ALL investments (NB: included or not in a benchmark or market portfolio) is equal to the sum of the future expected cash flow, discounted back to net present value, or***[26]*:*

$$\textbf{(11)} \qquad P_{\$} = \frac{CF(1+g)}{(1+k)} + \frac{CF(1+g)^2}{(1+k)^2} + \cdots + \frac{CF(1+g)^n}{(1+k)^n} + TV$$

where ***CF*** is the cash flow for each period; ***g*** is the growth rate of that cash flow; ***n*** the number of years; ***TV*** the terminal value, and (key) ***K*** the discount rate. No asset in the world, however defined, escapes this very simple axiom.

The key to solve for $P_{\$}$ is to know *ex ante* the applicable discount rate(k)**,** or what is also known as the Cost of Capital, Cost of Equity, etc. The widespread practice (to these days!) to determine (k) is once again to rely on the CAPM, or:

$$(k) = R_f + \beta * [\text{Equity Risk Premium}]$$

However, this most popular formula of the CAPM is where the weakness of the model appears the most obvious: there is no theory or acceptable model to determine the ERP *ex-ante*. This crucial variable remains to a large extend a matter of guestimate and "standard practice"[27].

Given the absence of any theoretical guidance, we solved the problem of quantifying (k) by developing an algorithm – ***the Lumen Global Value Compass*** [28]- that uses (a) live market data; (b) a three stage Dividend Discount construct; and (c) Reverse Optimization. Note that by construction, our (k) is equal to an Internal Rate of Return (**IRR**). The resulting variable allowed

[26] We never bought into the foolishness to use multiples such as P/E, especially at the global level or worse yet applied to Emerging Markets, our DNA. P/E is at best a Momentum indicator and is based on the senseless assumption that earnings are mean-reverting or, at the aggregate level, constant as a portion of National Income. See *the Lumen Global Value Compass Primer.*

[27] The standard practice has been to take the historical difference between the average return on bonds and the average return on equities. Ibbotson publishes estimates on ERP using historical data going back to 1926!

[28] Please contact Lumen Advisors to request a copy of the Lumen Global Value Compass Primer.

us to rank investments around the world across asset classes based on a single, unbiased, comparable metric representing implied value, or the rate of return implied in the market.

To be sure, there is no question that formula (11) above can be deconstructed with respect to $(\boldsymbol{k})$ in several different and ways alternative to the Lumen's Compass: we make no claim of having found the *Holy Grail of Finance*. However, what needs to be pointed out is that the challenge (or the trick!) in attempting to solve for $(\boldsymbol{k})$ is to formulate a methodology (algorithm) that produces an unbiased and comparable-across-markets metric ***without relying on flawed models (e.g. the CAPM), over-fitting exercises, forecast, or assumptions that are so specific and restrictive as to invalidate and contaminate the result.*** *T*he task is not just to solve for $(\boldsymbol{k})$, but to extract an unbiased, comparable, stationary estimate of the market implied expected return.

In fact, whether the calculated implied return is "right" or "wrong" is not even the point. Our methodology was most certainly never built or intended to be a forecasting tool of market returns (i.e., it was never intended to generate *a signal in the quant parlance!*). The system was conceived to be purely a measuring and ranking tool, similar to a meter, a measuring tape, a piece of string, etc. It is a tool to quantify with one single, consistent and unique metric what each market/asset is implying in terms of expected return. How the measurement is viewed – big, small, right, wrong - and used is up to the end "consumer"[29].

In addition, given that our method does not rely on a model or a theory, it is not really susceptible to be put through traditional back testing methodology (e.g. in sample or out of sample)[30]; indeed, the algorithm does not need a historical time series to come up with the solution. The method we developed just takes a "snap picture" via the help of a DDM and reverse optimization of what the market is implying at any specific point in time; and to that end it has turned out to be a very reliable (in terms of consistency) measure.

Nevertheless, and given that Finance is not *faith-based*, we did run various tests to determine if the $\boldsymbol{K_e}$ (or the excepted return) "explained" future performance. For example, we use the results of our system to rank and pick a portfolio of Emerging Market country indices and compared the results against the MSCI Emerging Market Index (MXEF); the outperformance over ten years of history is remarkable (see Appendix 1). In order to dig further, we applied *Bootstrapping* as a well-known statistical technique to these results and we found that less than 10% of the results were random, while a surprising 90% were explained by the $\boldsymbol{K_e}$. In the end, the test that matters is ***the pull to gravity of cash (or the basic law of finance):*** any and every asset will eventually be worth exactly the amount of cash flow it generates over the long term!

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[29] Lumen's Ke has been successfully used for example to rank Emerging Equity Markets and build an equally weighted portfolio of the eight cheapest markets (highest Ke). Lumen's EM Portfolio built using this method has outperformed the MSCI benchmark by over 5% over the last ten years; See Appendix 1.

[30] The DCF/DDM formula can be viewed as an identity if all three variables, CF, G and $\boldsymbol{K_e}$ are predetermined. If we use it as an equation to solve for one variable but make no assumption of the other three, then there should be nothing to test.

8. Application to the B&L Model: testing and results

Given these satisfactory results, the irrefutable concept/basic law at the base of the approach (the Discounted Cash Flow), the absence of restricting assumptions, and the consistency of the results, it occurred to us that the vector of market implied returns $\boldsymbol{K_{e.}}$ should be a valid unbiased and cleaner alternative to *the Prior Vector* of Equilibrium Excess Return $\boldsymbol{\Pi}$ in the B&L formula, at least one with less distortions and virtually no practical limitations.[31]

In fact, the approach used to quantify Lumen's $\boldsymbol{K_{e.}}$ is actually not too different from the one used by B&L: we used a three-stage Dividend Discount Model (firmly rooted in the most basic Law of Finance) instead of using the CAPM, and then applied the same Reverse Optimization approach to extract implied returns. The major differences are that we do not impose any restrictive assumptions, and most crucially *we do not need a well-defined market portfolio*. That is, and pertinent to the purpose of this paper, the main advantage of our method is that, from a purely practical point of view, the quantification of our $\boldsymbol{K_{e.}}$, or the market implied rate of return does not require and is totally independent from the identification of a "market portfolio" or benchmark (bound to be incomplete and subjective); it "works" with any generic investment universe and thus can be used across any investment mandate.

Thus, our version of the B&L model using the $\boldsymbol{K_e}$ is as follows:

$$\textit{(12)} \qquad E(R) = \left[(\boldsymbol{\tau S})^{-1} + \boldsymbol{P^T}\,\Omega^{-1}\,\boldsymbol{P} \right]^{-1} \times \left[(\boldsymbol{\tau S})^{-1}\,\boldsymbol{K_e} + \boldsymbol{P^T}\,\Omega^{-1}\mathbf{Q} \right]$$

with the $\boldsymbol{K_{e.}}$ defined as excess return (i.e. minus risk free rate).

Given the practitioner emphasis and approach of our exercise (i.e. the practical need to allocate assets in an efficient portfolio AND print attractive performance results), we tested our method, using as a ranking criteria the Sharpe Ratio (or excess return per unit of risk) calculated for the final optimized portfolio. We applied this test to four different optimized portfolios, determined in turn out of four investment universes defined as follows:

1. S&P. Based on the S&P500, this "market portfolio" was defined by choosing one large cap for each of the 10 sectors as defined by the GICS[32], plus the Barclays' Aggregate for bonds.
2. HSCI. Based on the Hang Seng Composite Index, the "market portfolio" was defined following the same methodology used for the S&P, plus one extra stock.

[31] Given our global and, in particular, Emerging Market (EM) DNA, we faced many of the practitioner dilemmas highlighted above in carrying out asset allocation exercises and constructing efficient portfolios. Indeed, it is precisely because of the EM nature (dynamic markets; little historical; far from some sort of efficiency; etc.) that the need to find a dependable asset allocator and portfolio construction tool became dominant. The focus of our algorithm was never intended to be a response to the B&L shortcomings.

[32] The Global Industry Classification Standard ("GICS") was developed by and is the exclusive property and a service mark of MSCI Inc. ("MSCI") and Standard & Poor's Financial Services LLC ("S&P").

3. World. The global “market portfolio” was constructed by using the MSCI global sectors indexes also as defined by the GICS plus the bond aggregate.
4. Institutional. The global institutional “market portfolio” was provided by an active institutional global asset allocator, thus representing a “real world” exercise.

In other words, we used equation **(12)** to define the *Posterior* and then used it as input in the MVO. The testing was based on monthly data collected from January 2006 to March 2016. All necessary inputs – Asset Classes, Market Cap, Variance-Covariance, Average Return, risk free rate, ERP, views, etc. - are all reported in the Appendices.

Using these four universes, we calculated the average monthly excess return, variance and Sharpe Ratio for each of the four “optimized” portfolio using as input to the MVO the vector of expected returns calculated with both the original B&L model using equation **(10)**, and Lumen’s version of the B&L or equation **(12)**, of course applying the same set of subjective views (see Appendices).

The results are listed in Table 1 below (note that these are monthly data):

Table 1 – Sharpe Ratios

	S&P	HSCI	World	Inst.
B&L	0.100	0.124	0.144	0.124
Ke	0.210	0.148	0.218	0.245

For the four optimized portfolios, Lumen’s method (denoted as Ke in Table 1) generated a much higher Sharpe Ratio, except for the HSCI where the Ratio, albeit still higher, is not markedly different.

The optimized portfolios also appear to be more stable with Lumen’s methodology (again, denoted by Ke), having a more balanced weight distribution across the assets as shown in tables 2 to 5. These results are particularly pleasing. Indeed, the distribution across assets in the B&L model is obviously highly dependent on the conditional distribution, which in turn is dependent on the subjective views. We purposely stretched the views in order to stress test our methodology. For example, and as can be seen in the Appendices, we assigned expected outcome to pairs of assets that are not consistent (bigger) with the covariance reported in the matrix **Σ.** This practically and statistically should have markedly distorted the allocation; from the results listed in the tables below it appears that our methodology “resisted” this stress test. Table 5 (the Institutional Universe) is a great example.

Table 2 - S&P

	Market Weights	B&L Weights	Ke Weights
amzn	13.8%	10.5%	
ko	9.2%	17.9%	29.9%
xom	16.8%	9.8%	7.8%
wfc	11.6%		
mrk	6.7%	8.1%	9.5%
fdx	2.0%		
googl	22.1%	30.1%	2.6%
dow	2.7%		
T	11.0%	16.2%	14.7%
duk	2.5%	7.4%	35.6%
aep	1.5%		

Table 3 - HSCI

	Market Weights	B&L Weights	Ke Weights
1093	0.8%	10.9%	
1211	2.9%		
902	2.2%	1.7%	32.2%
728	5.8%		
914	1.8%	9.4%	
939	33.0%		14.7%
2318	12.8%	38.8%	
857	29.1%		
168	0.8%		12.2%
753	1.9%		
bidu	8.9%	39.2%	40.9%

Table 4 - World

	Market Weights	B&L Weights	Ke Weights
W.Fin.	10.0%		
W.Ener	3.3%	4.5%	
W.H.Care	6.0%	76.9%	35.6%
W.IT	6.5%		
W.Mat	2.4%		
W.Util.	1.8%	2.1%	2.3%
W.Cons.D	7.0%		
W.Ind.	5.7%		
W.Cons.S	5.8%	0.1%	46.0%
W.Telco	2.2%	0.8%	16.1%
agg	49.4%	15.6%	

Table 5 - Institutional Universe

	Market Weights	B&L Weights	Ke Weights
US Large/mid Cap	16.4%		0.9%
World xUS Large/mid Cap	11.5%		
US Small Cap	2.5%	5.8%	
World xUS Small Cap	1.8%		
EM Equity	12.0%		
Global REIT (public and private)	5.9%		
US Govt Bonds	8.0%		40.7%
Ex-US Govt Bonds (USD)	14.8%		
US High Grade	6.2%		
Ex-US High Grade	12.7%		
US High Yield	1.4%		18.6%
Ex-US High Yield	0.4%		
Global Inflation-Linked (USD)	2.3%		
US Muni	1.5%		5.9%
EM External Sovereign (USD)	0.7%		
EM Local Sovereign (USD)	1.5%	94.2%	33.8%
EM Corporate (USD)	0.7%		

Source: Bloomberg, MSCI, CBRE Clarion, AON Hewitt, JP Morgan, BofA Merrill Lynch

Notes: US Large/mid Cap from MSCI US Index; WorldxUS Large/mid Cap from MSCI World Excluding US Index; US Small Cap from MSCI US Small Cap Index; WorldxUS Small Cap from MSCI World ex US Small Cap Index; EM Equity from MSCI Emerging Markets Index; Global Public REIT from S&P Global REIT Index USD (also a proxy for Global Private REIT); US Govt Bonds from BofA ML US Treasury Index; Ex-US Govt Bonds (in USD) from S&P/Citigroup International Treasury Bond Ex-US Total Return Index; US High Grade from iBoxx USD Liquid Investment Grade Index; Ex-US High Grade (in USD) proxied by BofA ML Euro Non-Sovereign Index; US High Yield from iBoxx USD Liquid High Yield Index; Ex-US High Grade from BofA Merrill Lynch Global Ex-US Issuers High Yield Constrained Index; Global Inflation-Linked (USD) from BofA Merrill Lynch Global Inflation-Linked Government Index; US Muni from S&P Municipal Bond Index; EM External Sovereign (USD) from JP Morgan EMBI Global Total Return Index; EM Local Sovereign (USD) from JP Morgan GBI-EM Broad USD Unhedged Index; EM Corporate (USD) from JP Morgan CEMBI Broad Composite Index

9. Conclusions

Despite the perception of considerable advances in the field of finance, there has been surprisingly very little evolution on the topic of asset allocation and portfolio construction. To that end, Markowitz, Modern Portfolio Theory, the Capital Asset Price Model and the Black-Litterman models continue to be the main reference point in the industry.

Of these models, B&L remains by far the most advanced and useful one as not only the statistical methodology applied mitigates the well-known problems of the MPT/MVO, but it also allows for the inclusion of subjective investor's views in the construction of the portfolio or asset allocation. This is no small achievement. In fact, by using subjective views, the B&L construct has taken back the task of portfolio construction and asset allocation away from the hand of the *mad statistician (i.e. the risk manager)* and put it back in to the hands of the active manager, admittedly one with a healthy quantitative inclination. Hence, B&L explicitly allows asset allocation and portfolio construction to be driven by expected returns as opposed to by dry and backward-looking risk, however elegantly or mathematically thoroughly defined.

Irrespective of this ingenious innovation, we have highlighted some problems, both theoretical and practical, with the initial part of the B&L model, or the part that derives the market implied expected return (the *Prior Vector* in Bayesian statistics parlance). Not only does the model rest entirely on the very restrictive assumptions at the base of the CAPM, but it also becomes somewhat impractical when it comes to define an all-encompassing investable universe – a crucial requirement according to the CAPM. In our view, the identification of market implied expected returns (i.e*. the Prior*) is a crucial starting point that should be unbiased, void of assumptions and forecast, and should certainly not be determined by flawed models. Also, it should be reasonably applicable to any group of assets, securities, investments, domestic and global; i.e. it shouldn't be dependent on first defining a hypothetical market portfolio that satisfies the (restrictive and theoretical) CAPM assumptions.

The methodology we propose in this paper, i.e. using the most basic law of finance (or a DCF) as the theoretical and practical framework to extract the implied return (for example, our $\boldsymbol{k_e}$), seems to us the most obvious place to start, particularly if implied returns are established without relying on theories, assumptions, and forecasts, but instead relying on good old cash flow (gravity). Put it simply, the approach we recommend can extract implied returns from ANY investable universe and/or subset of it without impairing the entire exercise. This somewhat original approach - original in terms of the origin of finance that is - coupled with the ingenious B&L statistical construct and in particular the inclusion of subjective views, appears to provide not only superior results as measured in terms of risk-adjusted returns (Sharpe Ratios), but also more stable, intuitive and therefore actionable portfolios.

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Appendix 1

Ke Application and Back Testing

Methodology (Out of Sample)

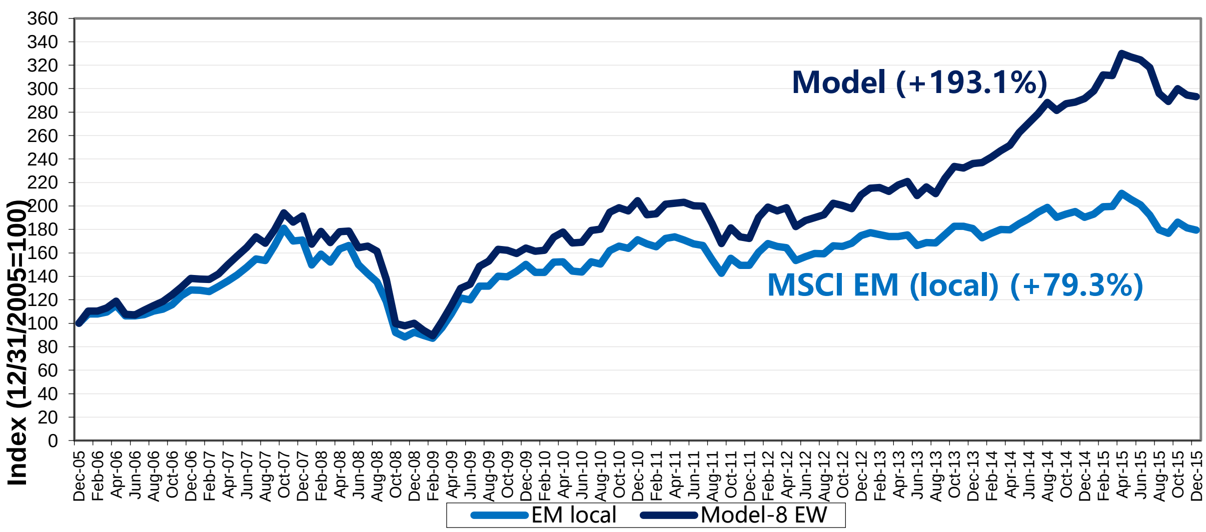
Model	8 country, equal-weighted
Description	Pick the 8 highest implied cost of equity (Ke)
Weighting	Equal-weighted
Rebalancing Freq.	Monthly; as of month-end
Universe	About 25 investable EM countries
Returns	Local Market Total Returns in local currency (incl. net dividends); before trading costs and fees

Comparative Statistics

SUMMARY	MSCI EM (Local)	Model
Av. Ann. Return	6.0%	11.4%
Annualized Vol	17.6%	19.3%
Downside Vol (ann.)	11.1%	12.6%
Info Ratio	0.34	0.59
Info Ratio (tracking error)		0.89
Tracking Error (ann.)		6.1%
Sortino Ratio	44.0%	81.2%
Alpha (monthly)		0.44%
Beta		1.04
Cumulative Performance	79.3%	193.1%

Outperformance vs. EM Index Ann. Avg. per year or Alpha (ann.)		5.3%

Emerging Market Stocks (MSCI) vs. Lumen Country Allocation
(Cumulative total returns, local terms, 12/31/2005 to 12/31/15)



*Gross Performance in local currency (in total returns including dividends) excluding fees and trading costs;
Methodology: Using the cheapest 8 countries based on Ke and local stock market indices (Bloomberg) based on a universe of around 25

Source: Bloomberg, Lumen Advisors; Model is gross hypothetical performance before fees and trading costs for the period 12-31-2005 to 12-31-2015

Appendix 2

Input for the S&P 500 Portfolio

Assets (in Bill. Of $)

	amzn	ko	xom	wfc	mrk	fdx	googl	dow	T	duk	aep	total
Mkt Cap.	$ 300	$ 200	$ 366	$ 252	$ 146	$ 43	$ 481	$ 59	$ 240	$ 55	$ 32	**$ 2,174**
Wheights	13.8%	9.2%	16.8%	11.6%	6.7%	2.0%	22.1%	2.7%	11.0%	2.5%	1.5%	**1.00**

Monthly Average Return - Beta - Implied Ke

	amzn	ko	xom	wfc	mrk	fdx	googl	dow	T	duk	aep
Av. Ret.	2.67%	1.02%	0.64%	1.00%	0.94%	0.71%	1.43%	1.15%	0.95%	0.91%	0.70%
Beta	1.10	0.57	0.60	1.20	0.71	1.23	1.10	2.04	0.56	0.32	1.41
Ke Monthly*	0.44%	0.89%	0.68%	0.73%	0.96%	0.48%	0.74%	0.86%	0.89%	0.75%	0.71%

"*Calculated using Lumen Global Value Compass©

Variance-Covariance Matrix

	amzn	ko	xom	wfc	mrk	fdx	googl	dow	T	duk	aep
amzn	0.0122										
ko	0.0015	0.0020									
xom	0.0008	0.0008	0.0023								
wfc	0.0008	0.0013	0.0009	0.0082							
mrk	0.0021	0.0010	0.0010	0.0008	0.0040						
fdx	0.0020	0.0014	0.0011	0.0035	0.0007	0.0059					
googl	0.0046	0.0010	0.0014	0.0013	0.0019	0.0019	0.0075				
dow	0.0031	0.0014	0.0015	0.0072	0.0015	0.0066	0.0037	0.0162			
T	0.0008	0.0011	0.0008	0.0007	0.0009	0.0015	0.0011	0.0016	0.0025		
duk	0.0004	0.0007	0.0004	0.0005	0.0010	0.0006	0.0008	0.0005	0.0008	0.0015	
aep	0.0032	0.0014	0.0015	0.0030	0.0017	0.0033	0.0026	0.0058	0.0016	0.0006	0.0052

Average Market Excess Return*	0.0043
Market Variance	0.0019
Lambda (ʎ)	1.1363

**Using 10 year UST*

Link Matrix P

Vector Q	amzn	ko	xom	wfc	mrk	fdx	googl	dow	T	duk	aep
0.0063	0	0	0	0	0	**-1**	0	0	**1**	0	0
0.0100	0	**1**	0	0	0	0	0	**-1**	0	0	0
0.0083	0	0	0	0	0	0	**1**	0	0	0	**-1**

View # 1= AT&T will outperform FedEx by 0.6% per month; View # 2 = Coca Cola will outperform Dow Chemical by 1% per month; View # 3 = Google will outperform AEP by 0.83% per month.

Optimal Portfolio Weights

	amzn	ko	xom	wfc	mrk	fdx	googl	dow	T	duk	aep
Market Weights	13.80%	9.20%	16.84%	11.59%	6.72%	1.98%	22.13%	2.71%	11.04%	2.53%	1.47%
B&L Weights	10.54%	17.91%	9.76%		8.07%		30.11%		16.24%	7.36%	
Ke Weights		29.86%	7.80%		9.51%		2.63%		14.65%	35.56%	

Appendix 3

Input for the Hang Seng Portfolio

Assets (in Bill. Of HK $)

	1093	1211	902	728	914	939	2318	857	168	753	bidu	total
Mkt Cap.	$ 41	$ 149	$ 115	$ 299	$ 93	$ 1,697	$ 659	$ 1,496	$ 42	$ 100	$ 457	$ 5,149
Wheights	0.8%	2.9%	2.2%	5.8%	1.8%	33.0%	12.8%	29.1%	0.8%	1.9%	8.9%	100.0%

Monthly Average Return - Beta - Implied Ke

Av. Ret.	0.0282	0.0375	0.0099	0.0084	0.0275	0.0122	0.0228	0.0061	0.0163	0.0179	0.0059
Beta	0.8265	0.9117	0.5681	0.6198	1.2180	0.9068	1.2590	0.9373	0.4466	0.9915	0.4833
Ke Monthly*	0.0085	0.0074	0.0149	0.0083	0.0086	0.0129	0.0083	0.0079	0.0080	0.0087	0.0073

"*Calculated using Lumen Global Value Compass©

Variance-Covariance Matrix

	1093	1211	902	728	914	939	2318	857	168	753	bidu
1093	0.0249										
1211	0.0072	0.0327									
902	0.0059	0.0017	0.0086								
728	0.0041	0.0028	0.0034	0.0076							
914	0.0110	0.0095	0.0067	0.0077	0.0230						
939	0.0057	0.0058	0.0042	0.0041	0.0080	0.0080					
2318	0.0086	0.0102	0.0059	0.0054	0.0123	0.0090	0.0175				
857	0.0063	0.0069	0.0044	0.0057	0.0091	0.0067	0.0086	0.0101			
168	0.0032	0.0026	0.0023	0.0023	0.0047	0.0031	0.0047	0.0029	0.0110		
753	0.0085	0.0087	0.0047	0.0054	0.0101	0.0063	0.0110	0.0059	0.0066	0.0207	
bidu	0.0050	0.0052	0.0024	0.0029	0.0053	0.0033	0.0053	0.0037	0.0015	0.0060	0.0054

Average Market Excess Return	0.0081
Market Variance	0.0082
Lambda (ʎ)	*0.4948*

Lamda calculated using the HSCI Index and the 10 year UST

Link Matrix P

Vector Q	1093	1211	902	728	914	939	2318	857	168	753	bidu
0.013	0	0	0	0	1	0	0	-1	0	0	0
0.008	1	-1	0	0	0	0	0	0	0	0	0
0.013	0	0	0	0	0	-1	1	0	0	0	0
0.008	0	0	1	0	0	0	0	0	0	0	0
0.008	0	0	0	-1	0	0	0	0	0	0	1

Optimal Portfolio Weights

	1093	1211	902	728	914	939	2318	857	168	753	bidu
Market Weights	0.8%	2.9%	2.2%	5.8%	1.8%	33.0%	12.8%	29.1%	0.8%	1.9%	8.9%
B&L Weights	10.9%		1.7%		9.4%		38.8%				39.2%
Ke Weights			32.2%			14.7%			12.2%		40.9%

Appendix 4

Input for the World Sectors plus Bond Aggregate Portfolio

Assets (in (000) of Bill US$)

	W.Fin.	W.Ener	W.H.Care	W.IT	W.Mat	W.Util.	W.Cons.D	W.Ind.	W.Cons.S	W.Telco	agg	total
Mkt Cap.	$ 7,285	$ 2,383	$ 4,408	$ 4,710	$ 1,744	$ 1,300	$ 5,078	$ 4,174	$ 4,192	$ 1,600	$ 36,000	$ 72,875
Wheights	10.0%	3.3%	6.0%	6.5%	2.4%	1.8%	7.0%	5.7%	5.8%	2.2%	49.4%	1.00

Monthly Average Return - Beta - Implied Ke

	W.Fin.	W.Ener	W.H.Care	W.IT	W.Mat	W.Util.	W.Cons.D	W.Ind.	W.Cons.S	W.Telco	agg
Av. Ret.	0.0018	0.0037	0.0078	0.0076	0.0050	0.0053	0.0078	0.0065	0.0091	0.0074	0.0069
Beta	1.3171	1.0661	0.6447	1.0138	1.3068	0.6456	1.0137	1.1142	0.6239	0.7365	1.3712
Ke Monthly*	0.0070	0.0072	0.0081	0.0071	0.0061	0.0076	0.0073	0.0073	0.0078	0.0084	0.0020

"*Calculated using Lumen Global Value Compass©

Variance-Covariance Matrix

	W.Fin.	W.Ener	W.H.Care	W.IT	W.Mat	W.Util.	W.Cons.D	W.Ind.	W.Cons.S	W.Telco	agg
W.Fin.	0.0043										
W.Ener	0.0027	0.0039									
W.H.Care	0.0019	0.0012	0.0014								
W.IT	0.0029	0.0023	0.0014	0.0027							
W.Mat	0.0036	0.0036	0.0017	0.0029	0.0047						
W.Util.	0.0018	0.0015	0.0010	0.0013	0.0018	0.0016					
W.Cons.D	0.0030	0.0021	0.0014	0.0024	0.0028	0.0013	0.0026				
W.Ind.	0.0033	0.0026	0.0015	0.0025	0.0033	0.0015	0.0026	0.0029			
W.Cons.S	0.0018	0.0013	0.0010	0.0013	0.0016	0.0011	0.0014	0.0015	0.0012		
W.Telco	0.0020	0.0018	0.0011	0.0016	0.0021	0.0013	0.0015	0.0018	0.0012	0.0018	
agg	0.0041	0.0031	0.0018	0.0030	0.0041	0.0018	0.0032	0.0036	0.0019	0.0023	0.0052

Average Market Excess Return	0.0030
Market Variance	0.0022
Lambda (ʎ)	0.6692

Lamda calculated using the MSCI World Index and the 10 year UST

Link Matrix P

	W.Fin.	W.Ener	W.H.Care	W.IT	W.Mat	W.Util.	W.Cons.D	W.Ind.	W.Cons.S	W.Telco	agg
0.0083	0	0	0	0	0	0	1	0	0	0	0
0.0083	0	1	0	0	0	0	0	0	0	0	0
0.0083	0	0	1	0	0	0	0	0	0	0	0

e absolute views, and all by ythe same amount.

Optimal Portfolio Weights

	W.Fin.	W.Ener	W.H.Care	W.IT	W.Mat	W.Util.	W.Cons.D	W.Ind.	W.Cons.S	W.Telco	agg
Market Weights	10.0%	3.3%	6.0%	6.5%	2.4%	1.8%	7.0%	5.7%	5.8%	2.2%	49.4%
B&L Weights		4.5%	76.9%			2.1%			0.1%	0.8%	15.6%
Ke Weights			35.6%			2.3%			46.0%	16.1%	

Appendix 5

Input for the Institutional Investable Portfolio

Assets in Bill US$

	US Large/mid Cap	World xUS Large/mid Cap	US Small Cap	World xUS Small Cap	EM Equity	Global REIT (public and private)	US Govt Bonds	Ex-US Govt Bonds (USD)	US High Grade	Ex-US High Grade	US High Yield	Ex-US High Yield	Global Inflation-Linked (USD)	US Muni	EM External Sovereign (USD)	EM Local Sovereign (USD)	EM Corporate (USD)	TOTAL
Mkt Cap ($ billions)	18,958	13,374	2,840	2,134	13,866	6,799	9,227	17,137	7,163	14,698	1,644	451	2,618	1,687	828	1,682	785	115891.5
Weights	16.4%	11.5%	2.5%	1.8%	12.0%	5.9%	8.0%	14.8%	6.2%	12.7%	1.4%	0.4%	2.3%	1.5%	0.7%	1.5%	0.7%	

Monthly Average Return - Beta - Implied Ke

	US Large/mid Cap	World xUS Large/mid Cap	US Small Cap	World xUS Small Cap	EM Equity	Global REIT (public and private)	US Govt Bonds	Ex-US Govt Bonds (USD)	US High Grade	Ex-US High Grade	US High Yield	Ex-US High Yield	Global Inflation-Linked (USD)	US Muni	EM External Sovereign (USD)	EM Local Sovereign (USD)	EM Corporate (USD)
Av. Ret.	0.0068	0.0042	0.0087	0.0055	0.0059	0.0067	0.0037	0.0037	0.0051	0.0038	0.0053	0.0072	0.0039	0.0040	0.0062	0.0043	0.0054
Beta to MXWD	0.8617	1.0757	1.0378	1.1194	1.2733	1.0467	(0.0732)	0.2473	0.1668	0.4251	0.4919	0.7138	0.2850	0.0428	0.3562	0.4043	0.3696
Ke Monthly	0.0068	0.0072	0.0055	0.0063	0.0075	0.0077	0.0011	0.0001	0.0027	0.0010	0.0066	0.0058	(0.0003)	0.0018	0.0049	0.0049	0.0048

Variance-Covariance Matrix

	US Large/mid Cap	World xUS Large/mid Cap	US Small Cap	World xUS Small Cap	EM Equity	Global Public REET	US Govt Bonds	Ex-US Govt Bonds (USD)	US High Grade	Ex-US High Grade	US High Yield	Ex-US High Yield	Global Inflation-Linked (USD)	US Muni	EM External Sovereign (USD)	EM Local Sovereign (USD)	EM Corporate (USD)
US Large/mid Cap	0.0019																
World xUS Large/mid Cap	0.0021	0.0028															
US Small Cap	0.0023	0.0025	0.0032														
World xUS Small Cap	0.0021	0.0029	0.0027	0.0033													
EM Equity	0.0024	0.0032	0.0029	0.0035	0.0046												
Global Public REET	0.0022	0.0026	0.0028	0.0027	0.0030	0.0038											
US Govt Bonds	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0001)	0.0001										
Ex-US Govt Bonds (USD)	0.0004	0.0007	0.0004	0.0007	0.0009	0.0007	0.0001	0.0006									
US High Grade	0.0003	0.0005	0.0003	0.0005	0.0006	0.0006	0.0001	0.0003	0.0004								
Ex-US High Grade	0.0007	0.0012	0.0008	0.0012	0.0014	0.0010	0.0000	0.0007	0.0003	0.0010							
US High Yield	0.0010	0.0012	0.0013	0.0013	0.0015	0.0015	(0.0001)	0.0003	0.0004	0.0005	0.0010						
Ex-US High Yield	0.0013	0.0019	0.0017	0.0021	0.0023	0.0018	(0.0001)	0.0006	0.0004	0.0010	0.0011	0.0017					
Global Inflation-Linked (USD)	0.0005	0.0008	0.0005	0.0008	0.0010	0.0008	0.0001	0.0005	0.0003	0.0006	0.0004	0.0007	0.0006				
US Muni	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0002			
EM External Sovereign (USD)	0.0007	0.0009	0.0008	0.0010	0.0012	0.0011	0.0001	0.0004	0.0004	0.0005	0.0006	0.0008	0.0004	0.0002	0.0006		
EM Local Sovereign (USD)	0.0007	0.0011	0.0009	0.0011	0.0015	0.0011	0.0000	0.0005	0.0003	0.0006	0.0005	0.0008	0.0004	0.0001	0.0005	0.0007	
EM Corporate (USD)	0.0007	0.0010	0.0009	0.0011	0.0012	0.0010	0.0000	0.0003	0.0004	0.0005	0.0007	0.0009	0.0004	0.0001	0.0006	0.0005	0.0007

Average Market Excess Return	0.002978
Market Variance	0.002364
Lambda (ʎ)	0.629795

Lambda based on MXWD index and 10y UST

Link Matrix P

Vector of Views Q	US Large/mid Cap	World xUS Large/mid Cap	US Small Cap	World xUS Small Cap	EM Equity	Global REIT (public and private)	US Govt Bonds	Ex-US Govt Bonds (USD)	US High Grade	Ex-US High Grade	US High Yield	Ex-US High Yield	Global Inflation-Linked (USD)	US Muni	EM External Sovereign (USD)	EM Local Sovereign (USD)	EM Corporate (USD)
0.005	0	0	0	0	0	0	-0.5	-0.5	0	0	1	0	0	0	0	0	0
0.005	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0.005	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0.005	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0
0.005	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	0
0.005	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0

Optimal Portfolio Weights

	US Large/mid Cap	World xUS Large/mid Cap	US Small Cap	World xUS Small Cap	EM Equity	Global REIT (public and private)	US Govt Bonds	Ex-US Govt Bonds (USD)	US High Grade	Ex-US High Grade	US High Yield	Ex-US High Yield	Global Inflation-Linked (USD)	US Muni	EM External Sovereign (USD)	EM Local Sovereign (USD)	EM Corporate (USD)
Market Weights	16.4%	11.5%	2.5%	1.8%	12.0%	5.9%	8.0%	14.8%	6.2%	12.7%	1.4%	0.4%	2.3%	1.5%	0.7%	1.5%	0.7%
B&L Weights			5.8%													94.2%	
Ke Weights	0.9%						40.7%				18.6%			5.9%		33.8%	